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# Neosperience Right-Time Personalization Technical Backgrounder and Computational Model Concepts

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# Neosperience - Right-Time Personalization Technical Backgrounder and Computational Model Concepts

## Summary

[Executive Summary](#)

[Neosperience RTP Basics](#)

[Personalization Engine](#)

[Advanced Features](#)

[Association with Custom Data Sources](#)

[Definition of a Standard Data Source \(SDS\)](#)

[Definition of a Custom Metric](#)

[Integration of a Customer Database with Data Coming from Social Networks](#)

[Introduction and previous work](#)

[Definition of the Problem and the Constraints on the Solution](#)

[Basics](#)

[Neosperience RTP Solution and Approach](#)

[Conclusion](#)

## **Executive Summary**

Neosperience Right-Time Personalization (RTP) is a powerful recommendation engine that is able to dynamically serve recommended products, text, or other content to users on different types of apps and websites (e.g., retail or media sites), based on a proprietary algorithm that delivers the most relevant and useful experience to the customer.

Neosperience RTP allows to increase customer satisfaction and drive sales by predicting customer's intent based on profile and behavioral data. This result is obtained by delivering the right message to each customer at the right time: highly relevant content that is based on the customer's context in the moment and is served to whatever device.

The right message to be delivered is chosen based on customer's context that is sensed, based on location or proximity to other objects and people, inferred from activities and social relations, or explicitly shown via social tagging, comments or wish-listed products.

This allows to understand product relationships, customer's behavior patterns and location, as well as to predict trends and deliver feedback as real-time decisions for customer next best action, predictive campaign analytics, uplift modeling, and customer-centric merchandising. In essence Neosperience RTP provides customers with an exact choice of products and services based on their profile and social behavior.

Typical usage scenarios are organizations with e-commerce needs that can use Neosperience RTP to drive product recommendations. Outside of e-commerce use cases, organizations have the ability to use our recommendation engine to drive contextual content across the app beyond just product recommendations. For example, organizations from the financial services and media and entertainment verticals can deliver content recommendations.

## Neosperience RTP Basics

Customer's attention is the ultimate scarce resource. Across industries marketing campaigns response rates are going down and customers are "banner blind" when organizations try to communicate more offers to them. Personalizing product information in real time and delivering them through multiple channels is the promise Neosperience RTP is delivering to marketers across a growing number of industries.

Personalization is all about understanding what customers need and presenting it to them in an easy, informative and compelling way. Choice overload lead your customers to decision-making paralysis, anxiety, and stress. And, in a culture that tells us that there is no excuse for falling short of perfection when your options are limitless, too much choice can lead to churn. While the explosion in choice has paradoxically become a problem for many organizations' customers, instead of an opportunity, making them feel worse, scientific evidence<sup>1</sup> shows that by eliminating irrelevant choices, you can greatly reduce the stress, anxiety, and busyness of your customers lives.

Neosperience RTP helps you increase your prospect database and conversion rate by reaching prospects and customers with the right message, at the right time, in the right context, on all their devices. This fundamental activity can be performed by supporting the customer goal-setting and decision making process, utilizing the massive information available within the customer's social networks, retrieved and analyzed by RTP, to offer him a trusted and reliable source of advice and inspiration.

The combination of real-time analysis with the capability to shape product selection and presentation, based on the pleasurable quality of customers' past experiences, allows you to wow your customers with an exact choice of relevant products, fitting precisely with their interests and passions, delivering optimized experiences on the web, Facebook, and mobiles, generating new revenue streams, unlocking the business value of your Facebook fan, prospect and customer base.

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<sup>1</sup> [http://en.wikipedia.org/wiki/The\\_Paradox\\_of\\_Choice:\\_Why\\_More\\_Is\\_Less](http://en.wikipedia.org/wiki/The_Paradox_of_Choice:_Why_More_Is_Less)

## Personalization Engine

Neosperience RTP allows to profile and deliver the best products by deeply analyzing your customers' interests graph: customers' behaviors, attitudes and motivation shape customer segments that describe the person who buys and uses your products. Such "personas" are connected to specific Neosperience RTP metrics, taking into account a broad range of demographics and psychographics data, as the basis of customer lifestyles, including the following interests categories, plus "likes" (i.e. your competitors' brands), tweets and social "check-ins" (i.e. Foursquare) and other context providers (i.e. local weather):

*Actor/Director, Movie, Producer, Writer, Studio, Movie Theater, TV/Movie Award, Fictional Character, Album, Song, Musician/Band, Musical Instrument, Playlist, Music Video, Concert Tour, Concert Venue, Radio Station, Record Label, Music Award, Music Chart, Book, Author, Book Store, Library, Magazine, Editor, TV Show, TV Network, TV Channel, Athlete, Artist, Public Figure, Journalist, News Personality, Chef, Lawyer, Doctor, Business Person, Comedian, Entertainer, Monarch, Teacher, Dancer, Politician, Government Official, Sports League, Professional Sports Team, Coach, Amateur Sports Team, School Sports Team, Restaurant/Cafe, Bar, Club, Company, Product/Service, Website, Cars, Bags/Luggage, Camera/Photo, Clothing, Computers, Software, Office Supplies, Electronics, Health/Beauty, Appliances, Building Materials, Commercial Equipment, Home Decor, Furniture, Household Supplies, Kitchen/Cooking, Patio/Garden, Tools/Equipment, Wine/Spirits, Jewelry/Watches, Movies/Music, Pet Supplies, Outdoor Gear/Sporting Goods, Baby Goods/Kids Goods, Media/News/Publishing, Bank/Financial Institution, Non-Governmental Organization (NGO), Insurance Company, Small Business, Energy/Utility, Retail and Consumer Merchandise, Automobiles and Parts, Industrials, Transport/Freight, Health/Medical/Pharmaceuticals, Aerospace/Defense, Mining/Materials, Farming/Agriculture, Chemicals, Consulting/Business Services, Legal/Law, Education, Engineering/Construction, Food/Beverages, Telecommunication, Biotechnology, Computers/Technology, Internet/Software, Travel/Leisure, Community Organization, Political Organization, Vitamins/Supplements, Drugs, Church/Religious Organization, Games/Toys, App, Local Business, Hotel, Landmark, Transit Stop, Airport, Sports Venue, Arts/Entertainment/Nightlife, Automotive, Spas/Beauty/Personal Care, Event Planning/Event Services, Bank/Financial Services, Food/Grocery, Health/Medical/Pharmacy, Home Improvement, Pet Services, Professional Services, Business Services, Community/Government, Real Estate, Shopping/Retail, Public Places, Attractions/Things to Do, Sports/Recreation/Activities, Tours/Sightseeing, Transportation, Hospital/Clinic, Museum/Art Gallery, Organization, School, University, Non-Profit Organization, Government Organization, Cause, Political Party.*

## **Advanced Features**

### **Association with Custom Data Sources**

Neosperience RTP Engine was developed to determine the evaluation of the user profile from standard data sources (SDS), in relation to a set of content of interest defined thanks to a set of metrics. The data source can be defined at will and constitutes a part independent from the rest of the personalization system. The Engine then operates considering, given a particular user profile, a set of metrics to the data source (eg, recent purchases, frequency of purchases, etc.), then putting these metrics in conjunction with other metrics evaluated in relation to the different data sources available, such as Facebook, Twitter, etc.

This approach allows the integration of customer and proprietary databases to take advantage of the available data in relation to the data extracted directly from other profiles and from Facebook as the most relevant social network today in the Western world.

The result is a calculation of relevance of content of interest, which is more typical in the case of the great basket of products for a given user at a given instant of time, based on her purchase history, but also of her interests and passions, identified either explicitly from what can be deduced from her profile, or implicitly, namely by analyzing the profiles and behaviors of her membership to social groups: family, friends, colleagues and acquaintances.

### **Definition of a Standard Data Source (SDS)**

The definition of a data source to the Engine can be performed by a specific API call. Neosperience RTP Engine exposes a series of REST API for creating a custom SDS: the system receives a description of the custom datasource using a JSON or XML configuration file, and associates that source to a specific application. The endpoint configuration provides to Neosperience RTP Engine also the URL to invoke for the extraction of user data and to update the same set of parameters to vary. This practice also provides insulation and protection in the security of the data related to the customer database, since such sensitive data never leave the system in which they are stored, but are simply drawn on the basis of predefined metrics. From the custom SDS only the result of this evaluation is therefore exported, which feeds the Neosperience RTP Engine for further processing. This way the protection of the data in the source system is ensured, thus addressing the basic issues of privacy and protection of user data.

## **Definition of a Custom Metric**

The system makes it possible, always using exposed APIs, the definition of custom metrics relevant to the specific SDS. This definition can take place at a basic or advanced. At the basic level the definition of the metric, such as the last purchases made by the user, is done by sending a configuration object to the system (JSON or XML). This object contains a composition of basic functions available in Neosperience RTP Engine and define the attributes of the custom SDS. This way, you can compose the valuation metrics of the user profile by combining elementary functional blocks in complete freedom, exposed by Neosperience RTP Engine. At an advanced level, if a complex metric cannot be expressed as a combination of the features available, we provide a Maven artifact that contains the implementation of the interfaces related to metrics, thus providing all the necessary Java code to the system for the evaluation of the complex metric. Although in our experience the integration of the standard configuration covers more than 90% of the possible metrics simply by using calls to the REST APIs, and we chose to implement this advanced integration mechanism to provide 100% coverage of the whole possible metrics space.

## **Integration of a Customer Database with Data Coming from Social Networks**

Since a distinctive peculiarity of Neosperience RTP Engine is its ability to relate data from different source profiles - for example, the customer database of a company and users of social networks - it is easy to understand the opportunity to increase a given data source with information from other sources. A typical case is to integrate and extend the customer database with data derived from Facebook users' profiles, once established the relation between the two profiles of the same individual.

This operation is easily achieved through the use of special REST APIs that allow to a certified identity, such as the backend or CRM system that takes care of the data extraction, to retrieve data for a given user. This operation is performed through a pull approach: the system calls Neosperience RTP Engine APIs to extract data related to a particular user, or to update the data extracted earlier. In case of need for synchronization operations of such data (push mode), you can achieve this simply by implementing SynchWorker Java interface, which is used by the system whenever an SDS changed, to trigger the update of the related data sources, and then of the customer database.



## Introduction and previous work

Before working on the model proposed we need to introduce a couple of fields on which are constructed actual solutions. The first one is the cosine similarity, defined as

$$sim(a, b) = cos(a, b) = \frac{(a \cdot b)}{|a||b|}$$

where the maximum similarity is obtained when vectors are parallel, reaching value zero. The operators  $|\cdot|$  and  $(\cdot, \cdot)$  defines the usual norm and inner product in the real N-th dimensional set. The strategy implemented by modern method of profiling is based on the extraction of informations from the user profile. Then the method analyzes the relevance ranking with a total parametric model; which infers a special variable, named CTR, click through rate.

The second field to introduce is the vector space model, based on the idea of concept. A concept is defined as a pair made of a keyword and a number (*keyword, c*), which represents an abstract cognitive classification. In particular, the c-value encodes the degree of belonging to the class identified by the keyword.

It is generally assumed c belongs to the real set, R, but there are several variations. The vector space model uses vectors of concepts to represent both the ad and the user. To do that, you are using a graph representation for the social network in which nodes can be of different types, each of them being connected to a node of linguistic description, named source node. From source node vectors are extracted concepts, operation CE (Concept Extraction). Such vectors are aggregated generating a single carrier for both to that for the individual:

$$u_x = U_x \Gamma_u$$

$$a_y = A_y \Gamma_a$$

Where  $U_x$ ,  $A_y$  are the matrices of the concepts for the user and for the ad, while  $\Gamma_{x,a}$  represents the weight vectors for the aggregation. These weights are unknown and will be estimated. Now the method uses a variation of the cosine similarity, previously described, between the two aggregated vectors. This variation has a special feature: return a non bounded value in order to weight the contributions. Finally, through a sigmoid function, the value of similarity is reported within the interval  $[0,1]$  and this value is interpreted as the probability of the individual taps on the ad (i.e. a promotional offer). In detail given  $u_x$ ,  $a_y$  as aggregates concept vectors, the comparison function is a variant of cosine similarity, in fact, the contributions of the two vectors in the similarity measure are weighed by a

transformation matrix  $W$ .

$$sim(u_x, a_y) = \frac{u_x^T W a_y}{\|u_x\| \|a_y\|} = \frac{\Gamma_u^T U_x^T A_y \Gamma_a}{\|U_x \Gamma_u\| \|A_y \Gamma_a\|}$$

The use of a weight matrix is an extension of the original measure based on some critical considerations: different concepts may have different capacity to distinguish the interests of the individual; also a concept in user space can meet a concept in the ad space, such as a Star Wars game satisfies both the concept of a videogame player, as well as of an individual who is passionate about Star Wars. Finally, concepts in users space may be different compared to those in the ad space.

Once computed the value of similarity, a sigmoidal function is applied to normalize the range [0,1]

$$CTR = \frac{1}{1 + e^z}$$

where

$$z = sim(u_x, a_y) + \gamma_0$$

Given this structure, the big step made has been to calculate the likelihood to be maximized in the search parameters. Data is collected through triplets  $(x_i, y_i, z_i)$ , where  $x_i$  is the user,  $y_i$  is ad  $z_i = 1$  as the dummy variable whose result is a click, or a tap, and is  $z_i = 0$  if the ad is not clicked or tapped. Given the sample let's maximize:

$$L = \sum_{i=1}^N \log g((2z_i - 1)(f(x_i, y_i) + \gamma_0))$$

The search strategy choice is similar to that used for the learning of a neural network; in particular it is based on the calculation of the gradients, the choice of a parameter of iterative learning and updating of the parameters. We can therefore obtain the following variations of parameters

$$\Delta\gamma_0 = r_0(z - p)$$

$$\Delta\Gamma_u = \frac{r_1(z - p)}{\|u_x\| \|a_y\|} U_x^T \left( I - \frac{u_x u_x^T}{\|u_x\|^2} \right) W a_y$$

$$\Delta\Gamma_a = \frac{r_2(z - p)}{\|u_x\| \|a_y\|} A_y^T \left( I - \frac{a_y a_y^T}{\|a_y\|^2} \right) W u_x$$

$$\Delta W = \frac{r_3(z - p)}{\|u_x\| \|a_y\|} u_x a_y^T$$

Where the variables  $r$  are the parameters of learning. The intuitive choice for these variables is  $r_0 < r_1 = r_2 < r_3$  as it seems that the number of parameters belonging to the third set is significantly larger than the others.

### Definition of the Problem and the Constraints on the Solution

The proposed problem is finding a model for sorting products that is based on considerations of the interests of an individual. This model must be based on the information retrievable from the social web. Since being extended to social networks other than Facebook is in current development, in this study we further restrict this to social networks. To exemplify, the type of information on which you rely concepts are based solely on Facebook likes. In this context, the units considered as the basis on which it is possible to build the computation are called metrics. A formal definition of the metric as a unit of the basis for the proposed reflections can be given starting from the idea of the concept.

### Basics

As said, first need is a definition of a quantity called metric. That is the brick on which all the model is built on. A metric is a binary concept that coincides with a specific user action within the social space.

The key features of a metric are two, the fact that it coincides with a specific action, i.e. a like on Facebook, and the fact that it is binary. Both features lead to a simplification: the first allows an easy extraction from the social space, while the second allows an easy data structures management. The disadvantage of this approach is that the types of such metrics

must be provided directly by the user of the model. By assumption that user is considered an expert in the representation choice. Even with this choice, it can be traced back to a vector space representation of fact, once fixed  $m$  metrics, the individual is viewed as a vector of  $m$  Boolean. A first difference compared to some of the proposed models is in the choice of using binary components instead of real numbers. This created the need to develop an ad hoc approach to this problem, in particular with regards to the processing of the vector representation for the product and the choice of the function of similarity.

A request in the creation can be to develop a model that can take into account any correlations between the internal metrics. This can be seen both as a generalization, in fact, thanks to sophisticated techniques, you can deduct these bonds to promote the results for the experienced user. The information in the representation of correlations can in fact be known to you from previous or external studies; we decided to deliver upon this need by providing the opportunity to enter and take into account this information.

Given the particular structure underlying the representation it must be decided the space belonging to the products, named *cards*, in a number equal to  $N$ . Assuming that this space should be as simple as possible in order to facilitate the user's task and based on the same working assumptions for the individual, assuming that the user has full knowledge of its products and therefore can provide all the information necessary for the completion of the scheme of solution, it is therefore required as objective the ordering of that list of products, based on the possible interests of the individual.

Although you can access the social network structure, in our case the only relevant information is the bond of friendship. You want access to the information concerning the individual to friends on Facebook in order to take advantage of intersections between the interests of the individual concerned and those of his friends. This dependency is not verified, it is assumed, by analyzing retrospectively the results for verification. Given this assumption, the models are required to explore patterns of change in the prediction based on this additional information contained in friendship. Given the diversities in the basic representation, it is difficult to fully exploit the ideas contained in the previous solutions. In addition, it is assumed that historical data are not available but that the model is required to be able to make predictions based simply on what can be seen in the current situation. The generalization of these methods to the case in which there are series of object it will be possible with ad hoc developments. As always happened, representation deficits will

necessarily have to be integrated by the further knowledge that will be entered as missing data, coming from other data sources. In summary, what Neosperience Right-Time Personalization is able to compute are models of dependence between metrics (MID), a representation of products in a space (PS), and a function of similarity between the products and individuals (customers), named relevance (REL), whose value will be used to sort metrics.

### Neosperience RTP Solution and Approach

By analyzing the ideas presented in the previous formulations of resolution you notice the idea of using probabilistic indices to give a relevance to the ad. What we did in RTP implementation has been to construct a model of this idea. Given the lack of historical data you will have to create a solid structure at the level of probabilistic interpretation. Therefore you will have to choose methods and distribution suited to give meaning to the model. The first thing to do is to search for the most suitable REL purposes. It is assumed that there exists a binary random variable, dependent on the individual and the product, which is the interest in this subject. The variable will be worth so either one if the individual is interested in the product concerned or zero if she is not.

$$I(X, Prod) = \begin{cases} 1, \\ 0, \end{cases}$$

Given this binary structure you may associate a Bernoulli distribution of this variable parameter  $p$ ,

$$I \sim Be(p)$$

which transcribes in probabilistic language the initial idea

$$\begin{cases} P(I = 1) = p \\ P(I = 0) = 1 - p \end{cases}$$

Given this structure, it creates immediately sought to identify the REL associated with the probability  $p$ . This index seems to be a good translation of the CTR associated with this case; in fact instead of inferring the probability of clicks (or taps), we infer the probability of interest of the individual. Moreover,  $p$  is also the value expected of the random variable, and as the predicted value is associated to the pair individual product. Since we assume a

dependence with respect to the individual and to the product,  $I = I(X, Prod)$ , the problem is expanding this index in a relationship of similarity. So it is needed to be able to decompose the index finding its dependencies from both the individual and the product. Is therefore given the individual as  $X$  belonging to  $R^m$  and, given the representation in the space of metrics, is a random variable that can assume a finite number of values. In fact, the possible values that can assume are the  $2^m$  realizations of the vector of booleans. Using the formulas of the conditional probability one can decompose the index chosen by tying to the individual  $X$ .

$$p = P(I = 1) = \sum_x P(I = 1|X = x)P(X = x)$$

This way the necessity to describe two aspects is born, one concerning the relationship between the individual  $X$  and the probability  $p$ , and the second related only to the description of the individual. Thanks to this decomposition the portion of the MID will come to be linked to this second part while the PS will influence specifically only the first part. The dependence of the product, which for now is subtended, will have to be specified when we will elaborate in the first term of the summation. This part, looked at individually, deals with the prediction of a random variable with binary response through a covariate vector. It is therefore traced to a regression problem, and given the particular choice of the response, binary, it is chosen to use logistic regression. This choice is quite classical when we want to well represent the data. Also an idea is often used as a scheme for prediction models CTR. Said  $x$  a realization of  $X$ , a logistic regression model imposes a particular link function, called logit.

$$\text{logit}(p) = x^T \beta$$

where  $p$  is the probability variable that we want to estimate and  $\beta$  is the vector of coefficients associated to the regression model.

The data which are expected to learn this model are tables in which for every product belonging to the deck, there are the descriptions of individuals and their assessment of interest in the product. Given this particular configuration of the data, the coefficients of the regression model can be interpreted as the representation of the product based on metrics. Indeed they are coefficients of importance of single metric against the probability of interest. The more they are great to have a much more positive value in the corresponding metric

increases the value of probability. This is crucial when you consider the constraint that the model must make predictions without historical data; in this case it is not possible to learn the coefficients from the data. It is however chosen to maintain this link function imposing that, in case of absence of data, there is a PS format by m-dimensional vectors, that represent these coefficients of importance. The values of this vector will, under the hypothesis of an expert, to be charged by the user of the model. Clearly, this force must be taken into consideration when I assess the negative aspects of the model, and if possible you will have to find solutions most appropriate to meet this requirement. Furthermore, through this forcing us `or` and you get a deterministic function, which has been lost throughout the analysis component of the variability in their statistical models. You sense the absence of this data, especially when you want to provide feedback on the operation of the model. In fact, no one can have any kind of perception of what the imputed values are valid and well-respected. Since you are working under assumptions of expert you will assume as a working hypothesis that these data are the most accurate possible.

In particular, a direct consequence is that it is assumed that the estimates obtained from a regression on the data tend to the value attributed by the expert, and then the limit of the two values match prediction. The deterministic function on which we will build will therefore be

$$P(I = 1|X = x) = \frac{e^{x^t \beta}}{e^{x^t \beta} + 1}$$

The coefficients of which has been discussed are indicated by the letter. These coefficients are chosen to be assigned in the range [-3,3] in order to give the user the possibility to make use of the concept of repulsion. This idea will be resumed in most points of the study; is a concept quite expensive and because it allows two effects extremely useful to the end user: on the one hand the ability to counteract the allocation of a product to an individual of which is known a priori not interest. This is the case for example of specifically feminine objects that you want to not end up in the basket of an individual who has given his being male. The other effect is the direction-some products, this is the case in which the user wants, for his choice, allocate certain products to an individual with certain specific characteristics, working with these coefficients one can achieve the effect. Because now the individual X, you want to be able to create a probabilistic model to describe it. The idea gained is to use a Bayesian approach to parametric inference on the metrics of the individual. With this method it is possible to describe accurately the dependencies between metrics, solving one of the

goals that we had prepared for them; also combines the possibility to read the individual as such, but correct values using those belonging to all the friends. Another important feature is that the model continues to operate even if there are no friends; which makes it robust to all possible situations that may arise in the social environment. Let  $Y_1, \dots, Y_a$  friends of the individual, since they are used as data for update the prior data, must be assumed that they also may be identically distributed random variable also with respect to  $X$ . An additional hypothesis imposes that this set is under condition of exchangeability.

The density chosen to describe the individual generic in this is a product of density Bernoulli, in fact since the metrics are binary variables seems a choice as less coherent. The dependencies are discharged over the parameters of the prior.

$$X, X_1, \dots, X_N | \theta \sim \prod_{i=0}^{m-1} Be(\theta_i)$$

$$\theta_1, \dots, \theta_m \sim \pi(\theta_1, \dots, \theta_m)$$

$$X, X_1, \dots, X_N \sim \prod_{i=0}^{m-1} Be(\theta_i)$$

$$\theta_1, \dots, \theta_m \sim \prod_{i=0}^m Beta(\alpha, \beta)$$

(ho aggiunto in suggerimento le formule perchè non riesco a inserire i pedici)

Where the former distribution identifies a product of bernoulli distributions while the latter the product of beta distributions. This conjugate choice meets the constraint over the posterior distribution to be computationally efficient. Some theoretical results can also support this choice to be a good one.

The prior/posterior must have some essential properties: first, to allow the parameters of two metrics of influence; In fact, you should be able to enter information on a priori connection between two metrics. It must also give the possibility to those who use the model to decide how much to leave to vary the prior using the data. This special feature is done giving special links between the parameters of the prior. Those links generate a Bayesian network in which means and probability are estimated by Monte Carlo algorithms implementing a pitfall effect. Back to the summation that gives the probability of interest can be seen that the dimension of the space is nearly  $2^m$  vectors of booleans. This exponential dimension raise the



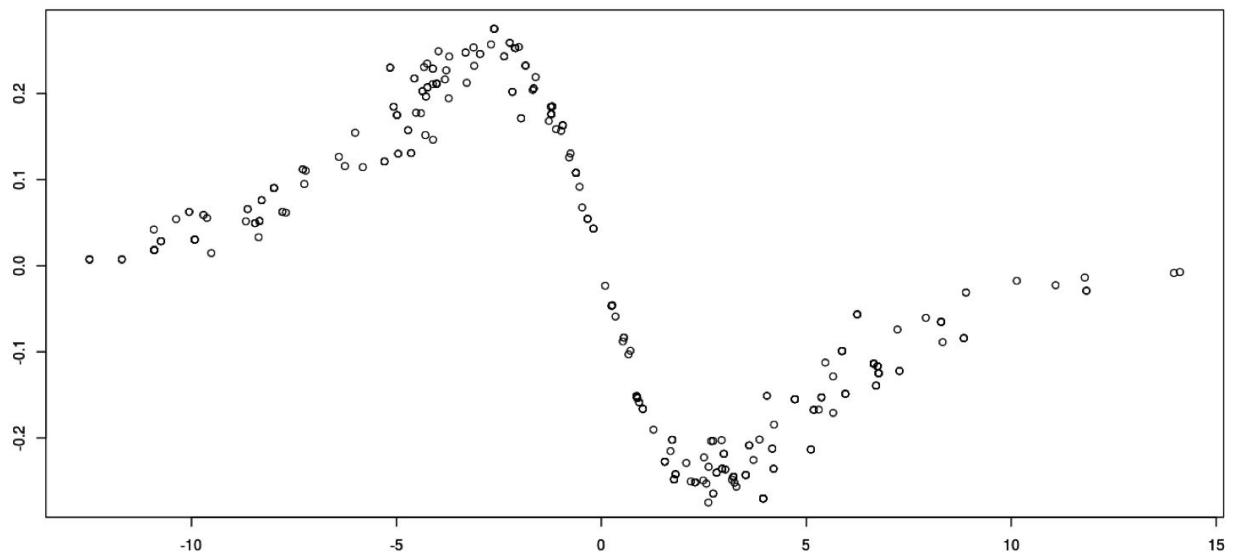
computational effort needed by this model to an impossible one. An approximation is needed. In this section we deal with the variance introduced by the approximation

$$E[g(X)] \simeq g(E[X])$$

Coming from the first equation and given the deterministic logit function

$$\sum_x P(I|X)P(X=x) = \sum_x g(x)P(X=x) = E[g(X)]$$

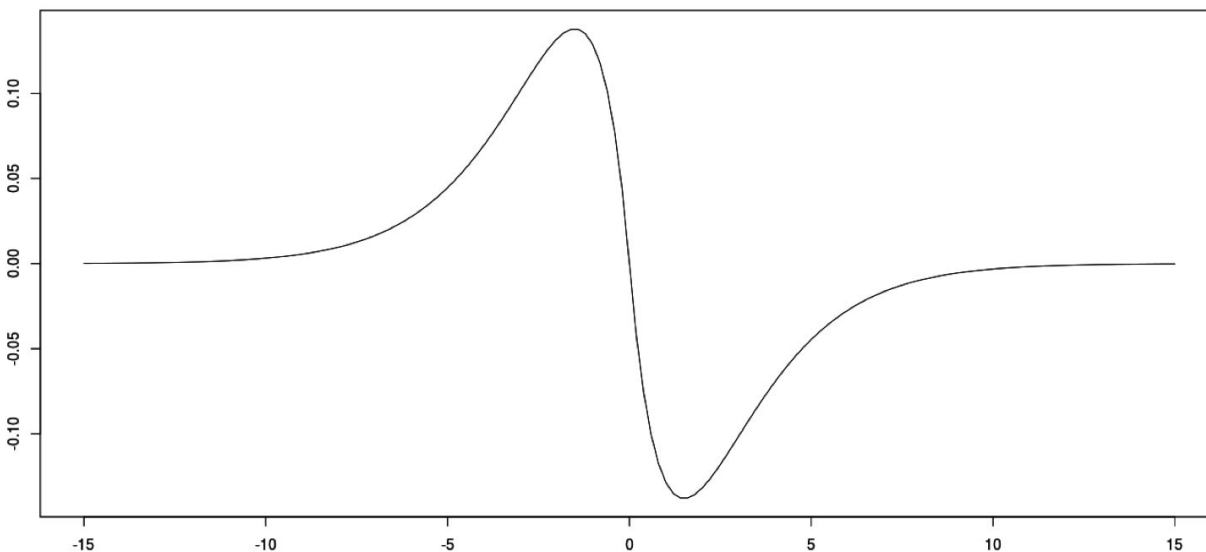
To analyze the error are generated in pairs two values of relevance; the first through the natural method, the second with the approximation. Were also observed in different quantities that intuitively could give indications on the size of the gap between the two estimates. One of these provides an extremely interesting correlation with the estimate error:



The graph shows the correlation; the essential aspects mainly concern the linearity in the neighborhood of zero, the apparent odd symmetry, the maximum and the fall nonlinear tails. The function so that it is sought to estimate this type of trend is a kernel range symmetrized odd

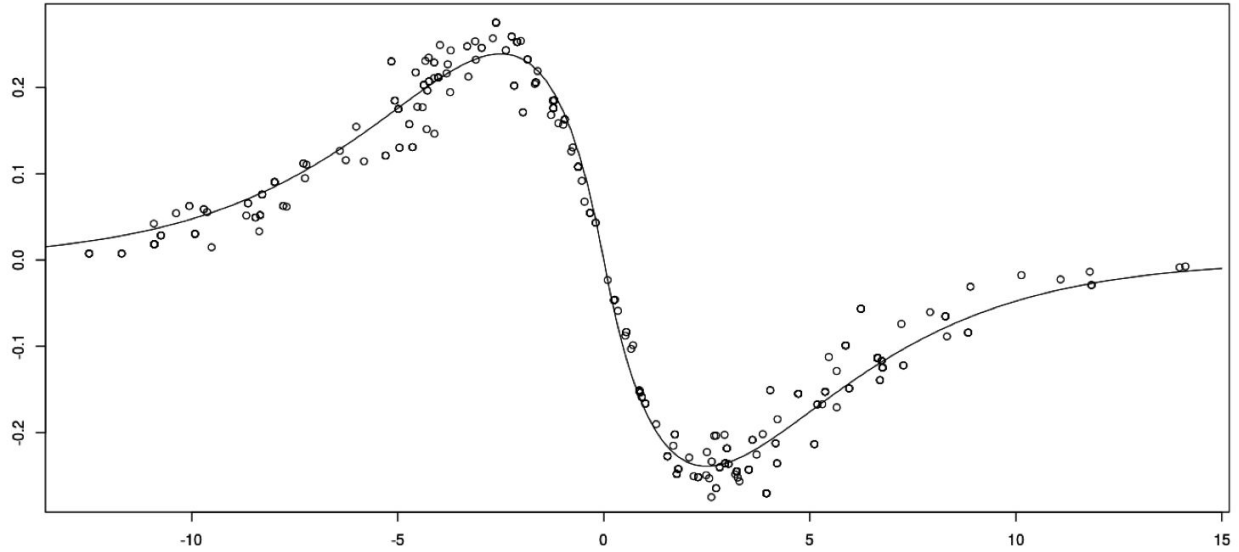
$$h(x) = c_1 x e^{-\frac{|x|}{c_2}}$$

a graph where possible, with random values of the parameters is represented in the figure.



It is known as the trends identified above are quite respected by this curve; it can be a great starting point for the analysis of the error. A basic method for estimating the two parameters of the kernel was to divide the chart into different curves according to different dimensionality to and then do an analysis for each. This method has the great advantage of being able to have a model for each curve, minimizing the error, but has the disadvantage of not being general; In fact, no one can say anything about the numbers which are at high beyond computational power. Also a lookup table is needed to compute the error for each pair  $(x, N)$ . The path we have chosen was to use all the data to estimate a single general model that has as covariates both the internal variable is the number  $N$ . This approach provides a general estimate for each pair  $(N, x)$  has one side of the fault surely commit an error greater than the former, on the other hand implies a very dangerous gamble, it is highly recommended to predict outside of its range of training and not just because you have information about that area. Unfortunately, this area is beyond any possible computational effort and, therefore, a general model can help estimating the error in that area. For example, if you wanted to be conservative, but at the same time give some more information you might see if somehow the error is monotone, in this case the best conservative estimate of the error is the last bend in my available in the training.

For us back to a case solved with the classical linear regression theory we applied to the data a logarithmic transformation. From that transformation we have inferred the parameters of a straight line and then inverse transform to plot the result.



## Conclusion

The result shown is an impressive correlation, without any comparable as to our knowledge as of today, between the considered variables. This outlines Neosperience Right-Time Personalization engine as the most reliable recommendation system based on both social (notably Facebook) and proprietary CRM data to date.